Warsaw University of Technology

Faculty of Civil Engineering

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BUILDING MATERIALS

LABORATORY

Concrete design: three equations method

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1. Aim of the task

The aim of the task is to learn the principles of concrete design by the three equations method and to use this method to design ordinary concrete for the indicated specification.

2. Theorethical bacground

2.1.Definitions according to PN-EN

Concrete – a material resulting from mixing cement, coarse and fine aggregate, water and any possible admixtures and additives, which obtains its properties as a result of cement hydration.

Concrete mix – completely mixed concrete components that are able to be compacted using the chosen method.

Hardened concrete - concrete that is solid and has reached a certain level of strength.

Concrete produced at the construction site – concrete produced at the construction site by the contractor for his own use.

Commodity concrete – concrete delivered as a concrete mix by a person or entity who is not a contractor.

Precast concrete product – a concrete product formed and maturing at a location other than its final location.

Ordinary concrete – concrete with a dry density greater than 2000 kg/m^3 but not exceeding 2600 kg/m^3.

Lightweight concrete – concrete with a dry density of not less than 800 kg/m^3 and not more than 2000 kg/m^3. This concrete is produced using only or partly lightweight aggregate.

Heavy concrete – concrete with a dry density greater than 2600 kg/m^3.

High-strength concrete – concrete with a compressive strength class greater than C50/60 for ordinary and heavy concrete and concrete with a higher compressive strength class than LC50 /55 for lightweight concrete.

Designed concrete – concrete whose required properties and additional features are given to the manufacturer, who are responsible for delivering concrete in accordance with the required properties and additional features.

Formula concrete – concrete whose composition and components to be used are given to the manufacturer responsible for supplying concrete with such a specific composition.

Cubic meter of concrete – the amount of concrete mix that, when compacted in accordance with the procedure given in EN 12350-6, occupies a volume of one cubic meter.

Admixture – a component added during the mixing process of a concrete mix in small amounts in relation to the weight of cement to modify the properties of the concrete mix or hardened concrete**.**

Additive – a fine-grained component used for concrete to improve certain properties or obtain special properties; usually added in quantities above 5% of cement; the additive can significantly modify the properties of both concrete mix and hardened concrete.

Aggregate – granular material used in construction; aggregate can be natural, artificial or recycled.

Cement – finely ground inorganic material, which – when mixed with water – gives a cement paste, setting and hardening as a result of hydration reactions and processes, and after hardening remains strong and durable, also under water.

Water/cement ratio – ratio of the effective water content to the mass content of cement in the concrete mix..

Characteristic strength – the value of strength below which may be 5% of the population of all possible strength determinations for a given volume of concrete.

2.2.Introduction do concrete mix design

The concrete design process is the appropriate selection of qualitative and quantitative components in such a way that both the concrete mix and the hardened concrete are characterized by assumed properties. The basic properties of the concrete mix are consistence and workability, while in the case of hardened concrete the requirements relate in particular to compressive strength, i.e. the appropriate class of compressive strength of concrete, as well as other features related to its intended use and application (e.g. freeze-thaw resistance, tightness).

The three equations method is an experimental-computational method for designing the composition of ordinary concrete mix. Its name comes from the number of equations (conditions), on which it is based - Bolomey's strength equation (strength condition), absolute volume equation (tightness condition) and consistence equation (consistence condition). In order to design the composition of the concrete mix by three equations, you can proceed in three ways:

1. Solve a system of equations as following:

$$f_{cm} = A_{1,2} * \left(\frac{C}{W} \mp 0{,}5\right)$$

$$\frac{C}{\varrho_C} + \frac{A}{\varrho_A} + W = 1000 \pm 2\%$$

$$W = C * w_c + A * w_k$$

2. Using nomograms (graphs) (Fig. 1-3), which were developed by the authors of the method (Prof. T. Kluz and Prof. K. Eyman) to facilitate the use of the three equations method. It requires calculation of aggregate mix water demand ratio (w_A), as well as ratio of average (mean) compressive strength and the corfficient $A_{1,2}$ (A_1 or A_2) depending on cement class and aggregate type ($\frac{f_{cm}}{A}$). Based on those values it is possible to read from the proper nomograms approximate masses of cement, aggregate and water per 1 m^3 of concrete mix. The masses of the components are read according to the indicator w_A from vertical lines or curves, for aggregate with a density of 2.650 kg/dm^3 or with density of 2.900 kg/dm^3. From the read out masses of components per 1 m^3, a test specimen is prepared with a volume sufficient to verify the properties of the concrete mix and hardened concrete, and then the final composition is determined.

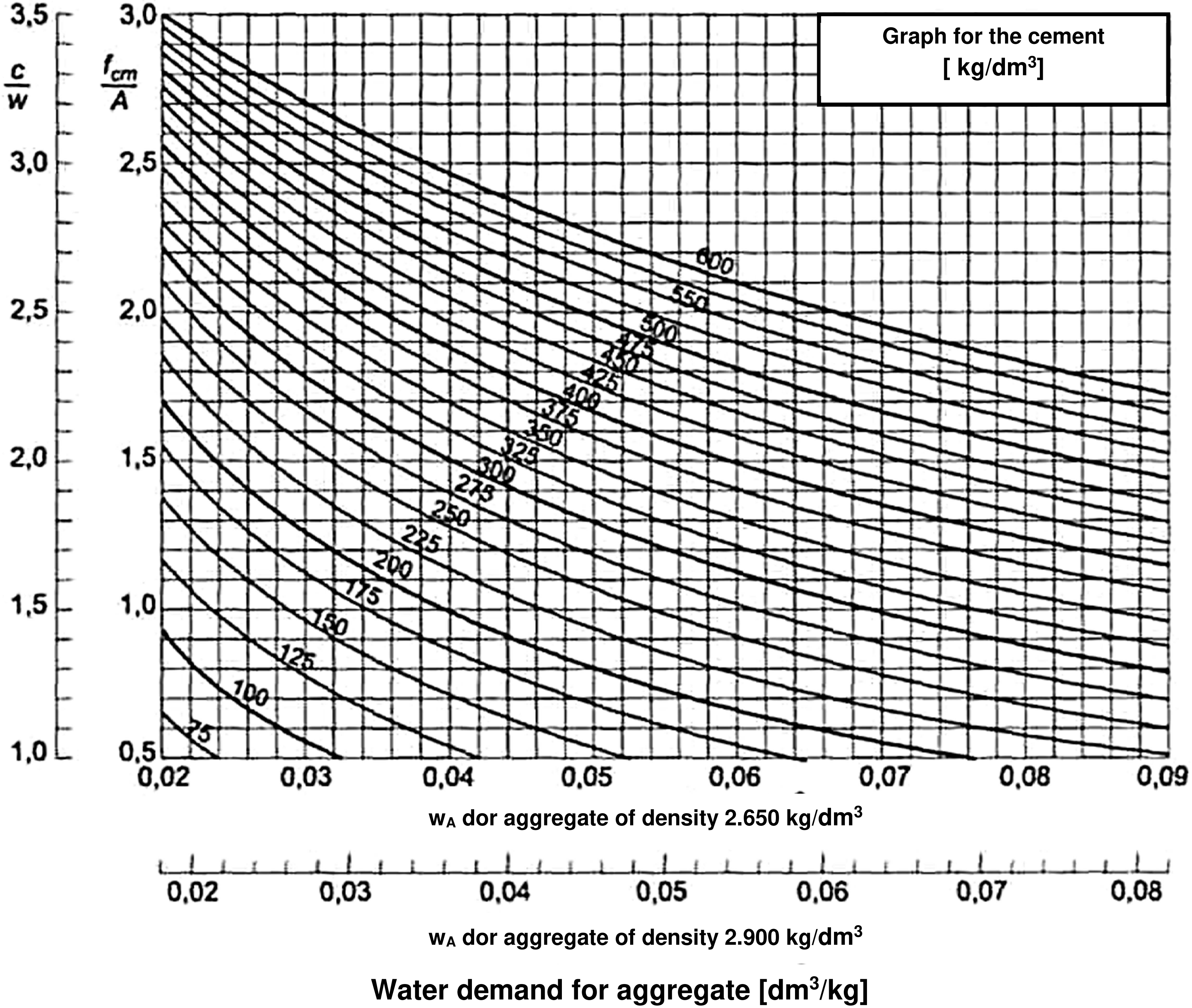


Fig.1. Nomogram graph of cement mass per 1 m^3 of concrete mix

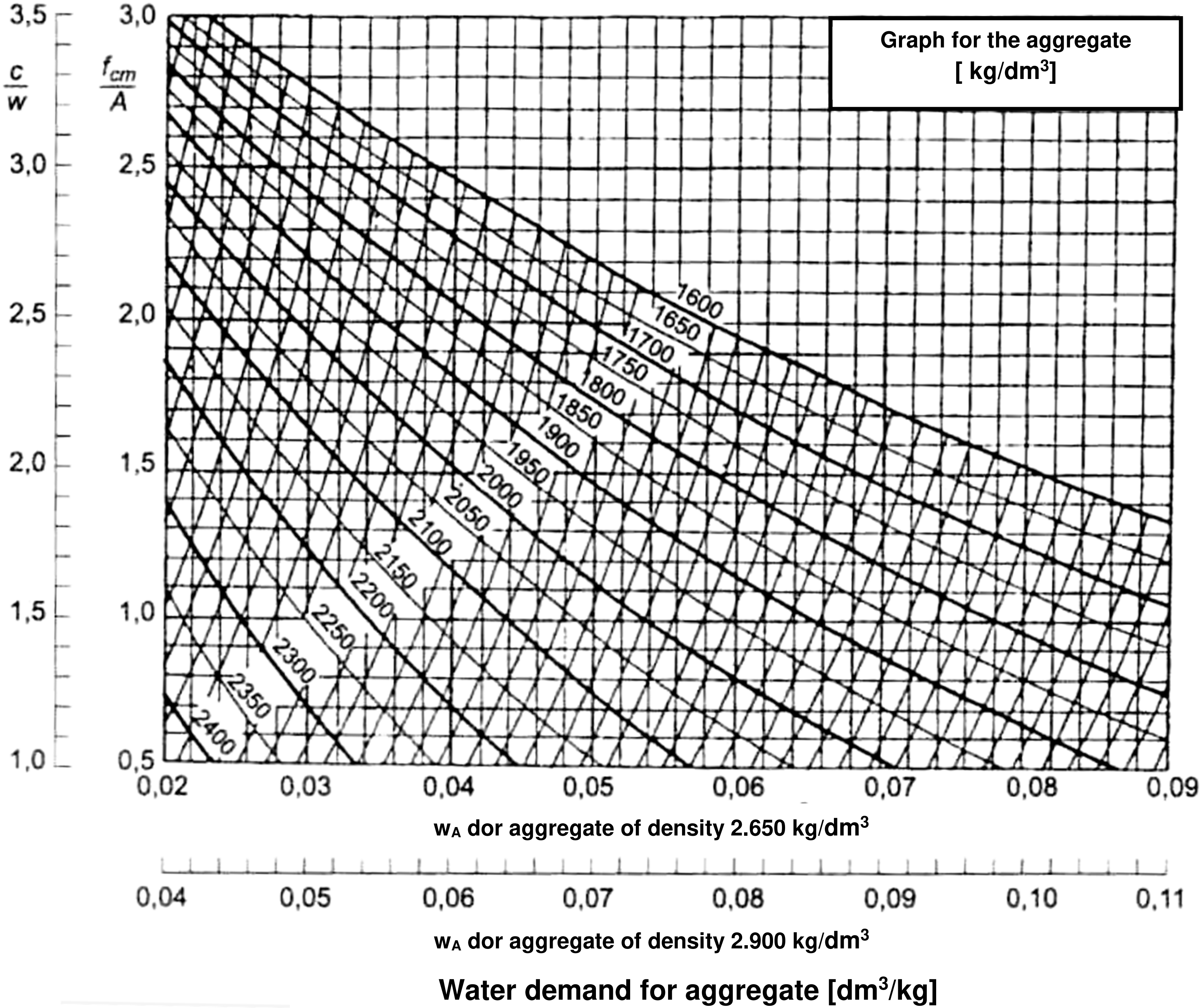


Fig.2. Nomogram graph of aggregate mass per 1 m^3 of concrete mix

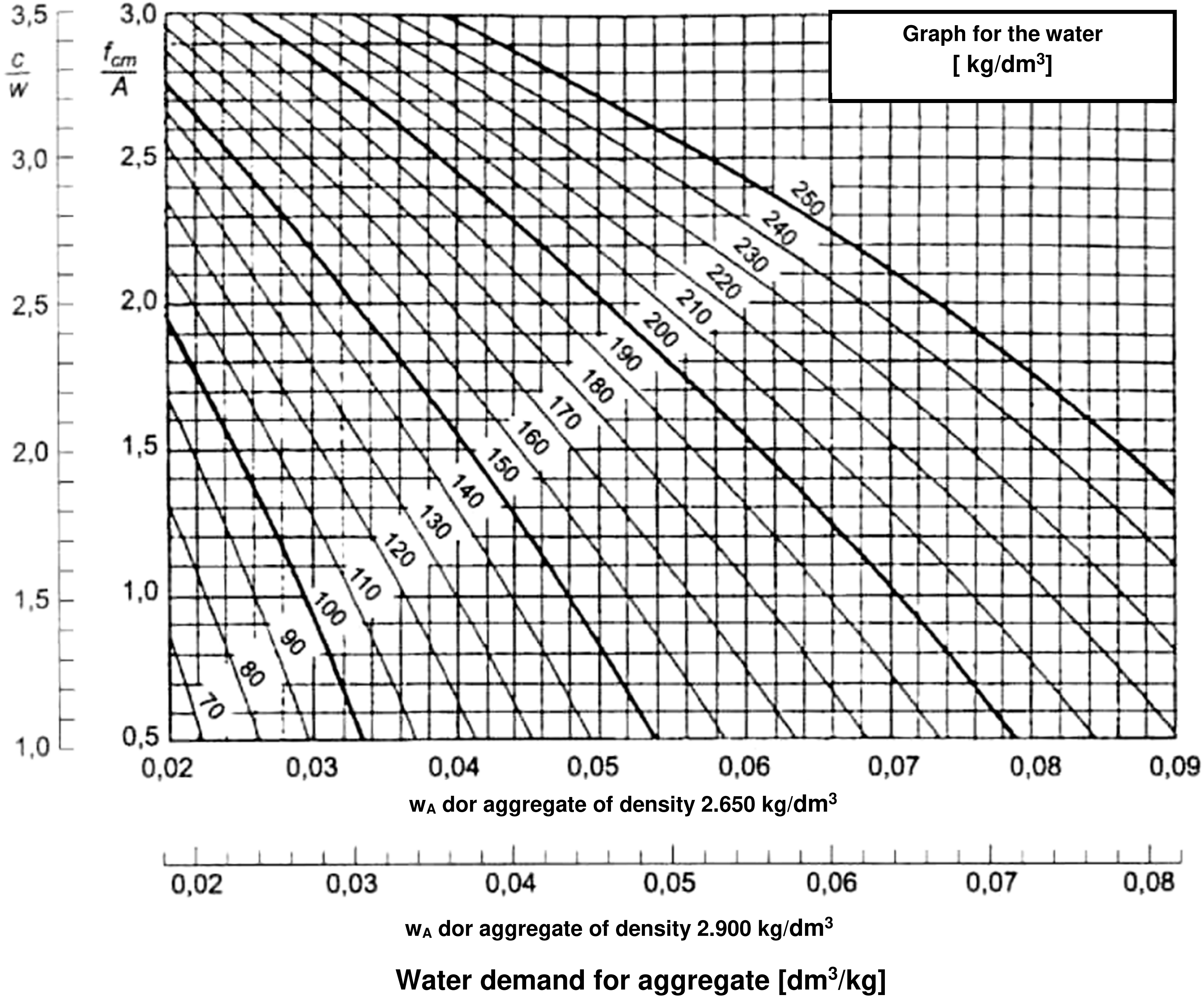


Fig.3. Nomogram graph of water mass per 1 m^3 of concrete mix

3. Using boards with ready, approximate compositions of concrete mix (Table 1). In order to read from them the masses of components per 1 m^3 of concrete mix, the cement-water coefficient (c/w), aggregate mix water demand ratio (w_A) and aggregate density (ρ_k) should be calculated.

Table1. Fragment of an example table with mass of components per 1 m^3 of concrete mix for c/w = 1.2 and aggregate of density ρ_k = 2.650 kg/ m^3 (all tables are available from the teacher)

Mass content for: *c/w* = 1.2 and ρ_k = 2.650 kg/m³				
w_A **[dm³/kg]**	***C* [kg]**	***W* [kg]**	***A* [kg]**	ρ_b **[kg/m³]**
0,026	101	84	2341	2526
0,027	104	87	2331	2522
0,028	108	90	2320	2518
0,029	111	93	2310	2514
0,030	114	95	2300	2509
0,031	118	98	2290	2506
0,032	121	101	2280	2502
0,033	124	103	2270	2497
0,034	127	106	2260	2493
0,035	130	109	2250	2489
0,036	134	111	2241	2486
0,037	137	114	2231	2482
0,038	140	117	2221	2478
0,039	143	119	2212	2474
0,040	146	122	2203	2471
0,041	149	124	2193	2466

3. Practical task

3.1. Designing the composition of ordinary concrete mix by three equations method

3.1.1. Materials and equipment

- Cement,
- Tap water,
- Natural aggregate of fractions: 0/2 mm, 2/4 mm, 4/8 mm and 8/16 mm,
- Scale,
- Plastic bowls,
- Metal spoons,
- Proper set for testing the concrete mix consistence (e.g. for Vebe test, slump test),

- Concrete cube molds of dimensions 15 x 15 x15 cm,
- Antiadhesive agent,
- Brush,
- Vibrator.

3.1.2. Task completion

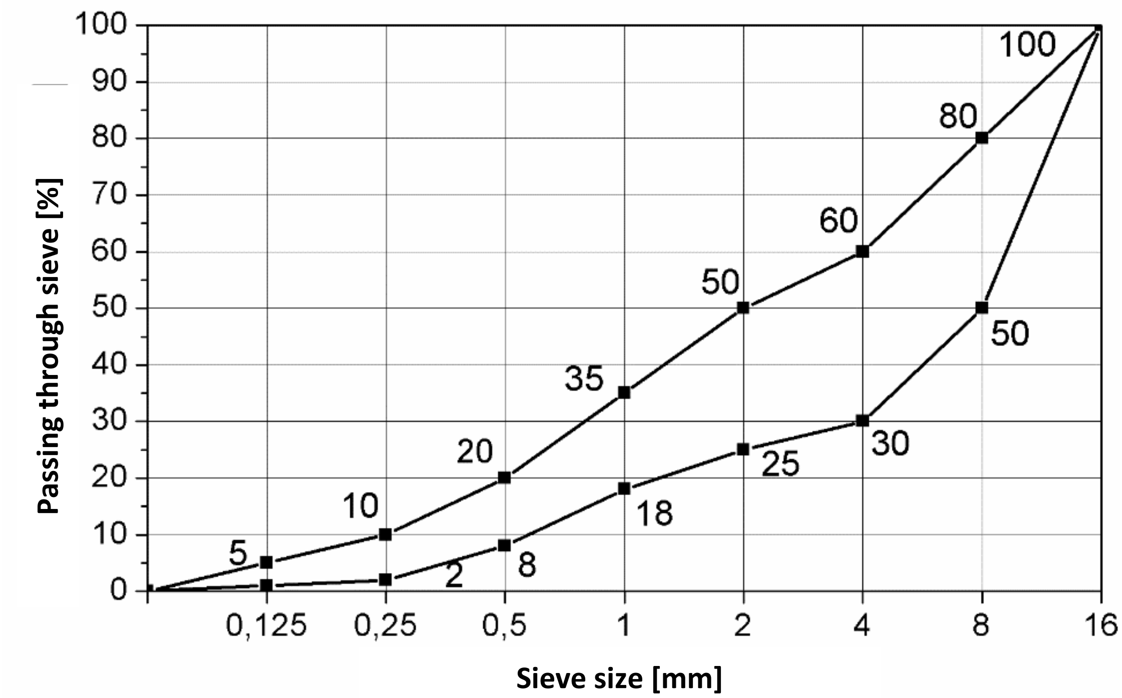
Designing the composition of ordinary concrete mix by three equations method consists of the following stages:

I. Preliminary assumptions for the concrete mix and concrete

Considering the purpose of the concrete and conditions indicated by the teacher – the type of structure (monolithic reinforced concrete structure, monolithic concrete structure, precast structure, massive structure, other types of structure), minimum structure size, reinforcement spacing, reinforcement cover thickness, etc., assumptions should be made regarding the designed concrete mix/concrete – purpose, exposure class, consistence class, strength class, maximum grain size class, type of specimens for testing the compressive strength, method of concrete mix compaction, concrete maturing conditions and others.

II. Qualitative selection, control and testing of concrete mix components

Selection of the type of materials (components) used to design the concrete mix - selection of the type of water, type and class of cement, aggregate type, aggregate fractions (taking into account the determined class of the maximum aggregate grain size - Fig. 4, Table 2). The components used to make the concrete mix should be thoroughly tested and meet the requirements of the relevant subject standards.



Fig, 4. Limiting curves of aggregate for concrete with an upper grain size of 16 mm recommended in the standard PN-B 06250

Table 2. The selected aggregate mix for ordinary concrete

Fraction [mm]	Content [%]
0/0,125	
0,125/0,0,25	
0,25/0,5	
0,5/1	
1/2	
2/4	
4/8	
8/16	
	***Σ* = 100%**

III. Quantitative selection of concrete mix components

(preliminary determination of the amount of components in the concrete mix and checking in a calculation and experimental way the correctness of the designed composition, making possible corrections to the composition and developing a working recipe).

To determine the composition of the concrete mix, follow these steps:

- Calculation of the cement-water ratio c / w using the Bolomey strength condition and the equation combining the designed average f_{cm} strength and characteristic strength f_{ck}:

$$f_{cm} = A_{1,2} * \left(\frac{c}{w} \mp 0.5\right)$$

$$f_{cm} = f_{ck} + 2\sigma$$

where: f_{cm} – average compressive strength [MPa], A_1 and A_2 - coefficients depending on the cement class and type of aggregate, C - mass of cement, W - water mass, f_{ck} - characteristic compressive strength [MPa], 2σ - compressive strength reserve (adopts equal to standard deviation if known, and if it is not known according to the standard, the value of 2σ = 3 ÷ 6 MPa is assumed);

Calculation of the aggregate mix water demand, taking into account the assumed consistency class and corresponding water demand coefficients (Table 3 and Table 4)

Table 3. Indices of water demand for the consistence determined in the slump test

Fraction [mm]	Water demand coefficients				
	V1	V2	V3	V4	V5
0/0,125	0,184	0,215	0,239	0,255	0,296
0,125/0,25	0,094	0,109	0,122	0,0137	0,151
0,25/0,5	0,064	0,076	0,084	0,095	0,112
0,5/1	0,045	0,053	0,058	0,065	0,077
1/2	0,033	0,039	0,043	0,048	0,058
2/4	0,025	0,029	0,032	0,037	0,044
4/8	0,020	0,023	0,026	0,029	0,034
8/16	0,015	0,018	0,020	0,023	0,027
16/32	0,013	0,015	0,016	0,018	0,022

Table 4. Calculation of the water demand of ordinary aggregate mix

Fraction [mm]	fraction content f_i [%]	Water demand coefficient for the fraction w_{Ai} [dm³/kg]	Water demand ratio for the fraction $f_i \cdot w_{Ai}$ [dm³/kg]
0/0,125			
0,125/0,25			
0,25/0,5			
0,5/1			
1/2			
2/4			
4/8			
8/16			
			$\Sigma = w_A$

- Determination of aggregate density of aggregate ρ_k;
- Reading the concrete mix composition from tables containing ready compositions per 1 m^3 and calculating the mass of individual aggregate fractions based on the selected in stage II aggregate mix (Table 5)

Table 5. The composition of the concrete mix read from the tables (aggregate divided into fractions)

<table>
<tr><th colspan="2">Component</th><th colspan="2">Mass [kg]</th></tr>
<tr><th colspan="2">Cement</th><td colspan="2"></td></tr>
<tr><th colspan="2">Water</th><td colspan="2"></td></tr>
<tr><th rowspan="8">Aggregate fraction [mm]</th><th>0/0,125</th><td rowspan="8"></td><td></td></tr>
<tr><th>0,125/0,25</th><td></td></tr>
<tr><th>0,25/0,5</th><td></td></tr>
<tr><th>0,5/1</th><td></td></tr>
<tr><th>1/2</th><td></td></tr>
<tr><th>2/4</th><td></td></tr>
<tr><th>4/8</th><td></td></tr>
<tr><th>8/16</th><td></td></tr>
</table>

- Experimental and computational verification of the correctness of the designed composition, which requires preparation of a test income with theoretical volume V_t sufficient to carry out the necessary tests of the concrete mix and concrete. To do this, weigh the aggregate from the coarse to the finest fraction and pour them into the bowl, then add cement and water in its entirety;
- Testing the consistence of the concrete mix using the Vebe test or Slump test and determining the class according to Table 6 or Table 7, if the consistence is in accordance with the assumed one, proceed to the next steps;

Table 6. Consistence class according to Vebe test

Class	**Vebe time [s]**
V1	30 ÷ 21
V2	20 ÷ 11
V3	10 ÷ 6
V4	5 ÷ 3

Table 7. Consistence class according to slump test

Class	**Slump [mm]**
S1	od 10 do 40
S2	od 50 do 90
S3	od 100 do 150
S4	od 160 do 210
S5	> 220

- Testing the real volume of the mix V_r, If $V_t \neq V_r$ the concrete mix components are adjusted to required volume.

$$C_2 = \frac{C}{V_{rz}} * 1000 \ \text{[kg]}$$

$$A_2 = \frac{A}{V_r} * 1000 \ \text{[kg]}$$

$$W_2 = \frac{W}{V_r} * 1000 \ \text{[kg]}$$

- Preparation of a 15 x 15 x 15 cm cubic sample for testing the compressive strength after 28 days of maturing;
- Equation of tightness (tightness condition) check:

$$\frac{C}{\rho_C} + \frac{A}{\rho_A} + W = 1000 \pm 2\%$$

where: C – cement mass per 1 m^3 of concrete mix [kg], ρ_C – cement density ($\rho_C = 3{,}1\ \mathrm{kg/dm^3}$), A – aggregate mass per 1 m^3 of concrete mix [kg], ρ_K – gęstość kruszywa ($\rho_k = 2{,}65\ \mathrm{kg/dm^3}$), W – water mass per 1 m^3 of concrete mix [kg].

If the tightness condition is not met, the calculated masses of the concrete mix components are adjusted (with constant value of c/w ratio);

- Checking if the designed composition of the concrete mix meets the requirements specified in the exposure class - requirements for maximum w/c ratio, minimum cement mass content, minimum compressive strength class (after testing the compressive strength);
- Providing a laboratory composition (the final composition per 1 m^3 of the concrete mix meeting the assumptions for aggregate in the air-dry state);
- Composition for works on the site:
  a) Composition per 1 m^3 of concrete taking into account the aggregate humidity (Table 8):

$$P_w = P\left(1 + \frac{w_d}{100}\right) \ \text{[kg]}$$

$$A_w = A\left(1 + \frac{w_g}{100}\right) \ \text{[kg]}$$

$$W_w = W - [(P_w - P) + (A_w - A)] \ \text{[kg]}$$

where: w_d – sand water content (humidity) [%], w_g – water content of aggregate of fraction 2/16 mm [%], P_w – corrected mass of sand [kg], P – mass of sand [kg], A_w –

corrected mass of aggregate of fraction 2/16 mm [kg], A – masa of aggregate of fraction 2/16 mm [kg], W_w – corrected water mass [kg], W – water mass [kg].

Table 8. Composition per 1 m³ of concrete mix taking into account the aggregate humidity

Component		Mass [kg]
Cement		
Water		
Aggregate	**0/2 mm**	
	2/4 mm	
	4/8 mm	
	8/16 mm	

b) Calculation of the mass of components per one concrete mixer or mixer, taking into account the aggregate moisture (Table 9):

Calculation of the capacity of the mixer:

$$V_r = V_t * \alpha$$

where: V_{rz} – mixer real volumetric capacity, V_t – theoretical mixer volume (data given by teacher), α – expansion coefficient equal 0,85

Calculation of the components masses per one mixer:

$$C_r = C * \frac{V_{rz}}{1000} \text{ [kg]}$$

$$W_r = W * \frac{V_{rz}}{1000} \text{ [kg]}$$

$$P_r = P * \frac{V_{rz}}{1000} \text{ [kg]}$$

$$A_r = A * \frac{V_{rz}}{1000} \text{ [kg]}$$

where: C_r – corrected cement mass [kg], C – cement mass [kg], P_w – corrected mass of aggregate 0/2 mm [kg], P – mass of aggregate 0/2 mm [kg], A_w – corrected mass of aggregate 2/16 mm [kg], A mass of aggregate 2/16 mm [kg], W_w – corrected water mass [kg], W – water mass [kg].

Table 9. Composition per one concrete mixer volume, taking into account the aggregate humidity

Component		Mass [kg]
Cement		
Water		
Aggregate	0/2 mm	
	2/4 mm	
	4/8 mm	
	8/16 mm	

c) Calculation of the mass of components taking into account dosing of cement with full bags and aggregate humidity:
The composition of the concrete mix should be recalculated by rounding the mass of cement so that it is divisible by the mass of the cement bag (25 kg) and given in Table 10.

Table 10. Composition per one concrete mixer, taking into account the aggregate humidity and cement dosing with full bags

Component		Mass [kg]
Cement		
Water		
Aggregate	0/2 mm	
	2/4 mm	
	4/8 mm	
	8/16 mm	

4. Laboratory report

The laboratory report should include:

I. Subject, aim and scope of research (containing basic information about tested materials/products, test methods, requirements),
II. Tests results with proper units (results obtained in the laboratory prepared in the indicated manner, e.g. put in the proper tables),
III. Conclusions (bulleted statements formulated based on the results obtained),
IV. Bibliography (list of references to the literature or www used to prepare the report).